

Topics : Work, Power and Energy, Circular Motion, Center of Mass, Relative Motion

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.5

(3 marks, 3 min.)

M.M., Min.

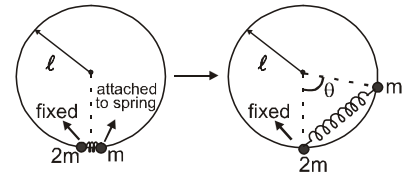
[15, 15]

Comprehension ('-1' negative marking) Q.6 to Q.9

(3 marks, 3 min.)

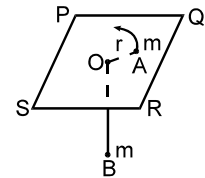
[12, 12]

1. The string is now replaced by a spring of spring constant k and natural length ℓ . Mass $2m$ is fixed at the bottom of the frame. The mass m which has the other end of the spring attached to it is brought near the mass $2m$ and released as shown in figure. The maximum angle θ that the spring will subtend at the centre will be : (Take $k = 10 \text{ N/m}$, $\ell = 1 \text{ m}$, $m = 1 \text{ kg}$ and $\ell = r$)



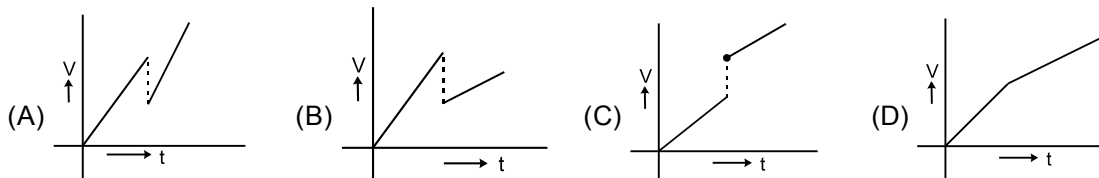
- (A) 60° (B) 30° (C) 90° (D) None of these

2. In the figure PQRS is a frictionless horizontal plane on which a particle A of mass m moves in a circle of radius r with an angular velocity ω such that $\omega^2 r = g/3$. Another particle of mass m is tied to A through an inextensible massless string. O is the hole through which string passes down to B. B can move only vertically. The tension in the string at this instant will be:

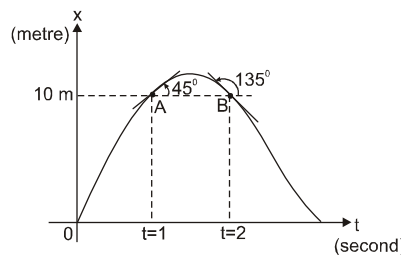


- (A) $mg/3$ (B) $2mg/3$ (C) $mg/6$ (D) none

3. Two balls of same mass are released simultaneously from heights h & $2h$ from the ground level. The balls collide with the floor & sticks to it. Then the velocity-time graph of centre of mass of the two balls is best represented by :



4. Displacement-time curve of a particle moving along a straight line is shown. Tangents at A and B make angles 45° and 135° with positive x-axis respectively. The average acceleration of the particle during $t = 1$, $t = 2$ second is :

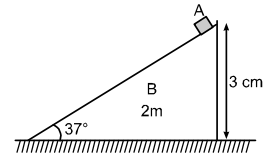


- (A) -2 m/s^2 (B) 1 m/s^2
(C) -1 m/s^2 (D) zero



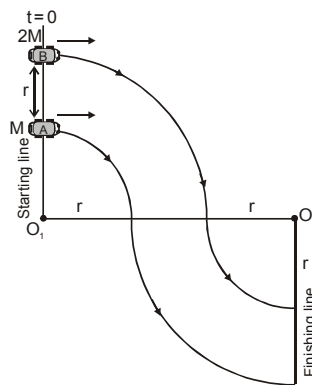
5. A particle A of mass m is situated at highest point of wedge B of mass $2m$ is released from rest. Then distance travelled by wedge B (with respect to ground) when particle A reaches at lowest position. Assume all surfaces are smooth.

- (A) $4/3$ cm (B) $8/3$ cm
(C) $2/3$ cm (D) none of these

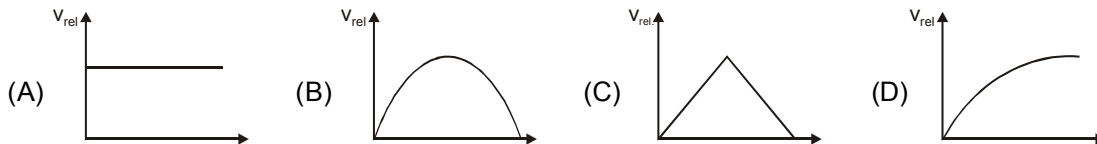


COMPREHENSION

Two racing cars 'A' and 'B' having masses ' M ' and ' $2M$ ' respectively start running from the starting line on a horizontal plane. Both cars 'A' and 'B' have same speed ' V ' which is constant through out the journey. The track of the two cars are the arcs of concentric circle having centres O_1 and O_2 as shown in figure with data. The friction coefficient of the two cars with the road is same. There is a finishing line at the end of the arc. Using these informations solve the following questions.



6. Graph between the magnitude of relative velocity of the two car and time is :



7. The magnitude of relative acceleration of two cars when car 'A' just reaches the end of circular arc of radius ' r '.

- (A) 0 (B) $(\sqrt{5-2\sqrt{2}}) \frac{V^2}{2r}$ (C) $\sqrt{2} \frac{V^2}{r}$ (D) $\frac{V^2}{r}$

8. The time interval during which the two cars have same angular speed :

- (A) Always along the motion (B) Never
(C) $\frac{\pi r}{2V}$ (D) $\frac{\pi r}{4V}$

9. Which of the following statements is **incorrect**.

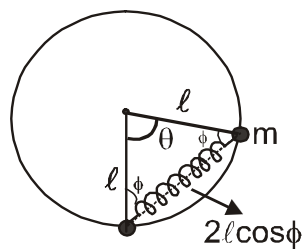
- (A) Both car reaches the finishing line at same time.
(B) Frictional force acting on the cars is directed towards centre whenever it acts
(C) Frictional force have same magnitude for two car 'A' and 'B' during the trip.
(D) None of these

Answers Key

1. (A) 2. (B) 3. (B)
 4. (A) 5. (A) 6. (B)
 7. (B) 8. (C) 9. (C)

Hint & Solutions

1. Length of spring at maximum = $2l \cos\phi$
 \therefore Extension is $x = (2l \cos\phi - l)$
 Now initial potential energy of the spring is converted into final PE of spring and gravitational PE.



$$\therefore \frac{1}{2} k l^2 = \frac{1}{2} k (2l \cos\theta - l)^2 + mg (l - l \cos\theta)$$

Putting values

$$\frac{1}{2} \times 10 \times 1^2 = \frac{1}{2} \times 10 (2 \cos\phi - 1)^2 + 10$$

$$(1 - \cos\theta)$$

$$\therefore \theta = \pi - 2\phi$$

$$5 = 5 (2 \cos\phi - 1)^2 + 10 (1 + \cos 2\phi)$$

$$1 = (4 \cos^2\phi + 1 - 4\cos\phi) + 2(1 + 2\cos^2\phi - 1)$$

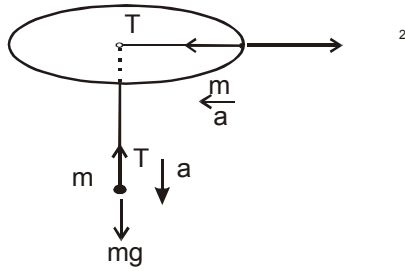
$$8 \cos^2\phi = 4 \cos\phi$$

$$\therefore \cos\phi = \frac{1}{2}$$

$$\therefore \phi = 60^\circ$$

$$\theta = 60^\circ$$

2. $T - m w^2 r = ma$



$$T - m \left(\frac{g}{3} \right) = ma \quad \longrightarrow \quad (1)$$

$$mg - T = ma \quad \longrightarrow \quad (2) \quad T - \frac{mg}{3} = mg$$

$$-T$$

$$\Rightarrow 2T = 4 mg / 3$$

$$\Rightarrow T = 2 mg / 3 \quad \text{Ans. (B)}$$

3. **(B)** As both the balls are released simultaneously, at any instant before the lower balls reaches the ground both have the same velocity ; $v = gt$ i.e. 'v' vs. 't' is a straight line graph.

$$V_{CM} = \frac{mv(t) + mv(t)}{2m} = v(t) ; v(t) \text{ being the}$$

instantaneous velocity.

Just after the lower ball strikes ground and comes to rest :

$$V_{CM} = \frac{mv(t)}{2m} = \frac{v(t)}{2}$$

i.e. the velocity suddenly drops to half its value.

Hence graphs (A) & (B) are chosen.

After collision :

$$a_{CM} = \frac{m(g) + m(0)}{m + m} = \frac{g}{2}$$

i.e. the slope (of v-t curve) should decrease to half.



$$4. \quad a_{av} = \frac{V_f - V_i}{\Delta t} = \frac{(\text{Slope at B}) - (\text{Slope at A})}{1s}$$

$$= \frac{-1-1}{1} = -2 \text{ m/s}^2$$

$$5. \quad \Sigma m \Delta r_{cm} = m_1 \Delta r_1 + m_2 \Delta r_2$$

$$= (m + 2m)(0) = m(x - 4) + 2m(x)$$

$$\Rightarrow x = \frac{4}{3} \text{ cm.}$$

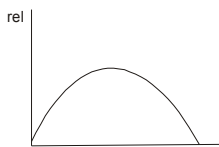
6. At any moment relative velocity $v_{rel} = v_A - v_B$
It has same magnitude but different direction so

$$v_{rel} = \sqrt{v^2 + v^2 + 2v^2 \cos(180 - \theta)}$$

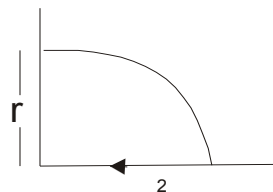
$$= \sqrt{2v^2(1 - \cos \theta)}$$

$$v_{rel} = \sqrt{2}v \sin \frac{\theta}{2}$$

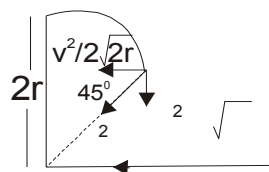
θ change with time t



7. For A



For B



$$= a_{rel} = \left\{ \left(\frac{v^2}{r} - \frac{v^2}{2\sqrt{2}r} \right) \hat{i} - \frac{v^2}{2\sqrt{2}r} \hat{j} \right\}$$

$$= |a_{rel}| = \frac{v^2}{2\sqrt{2}r} (2\sqrt{2} - 1)^2 + 1^2$$

$$\begin{aligned}
&= \frac{v^2}{2r} \sqrt{\frac{8+1-4\sqrt{2}+1}{2}} \\
&= \frac{v^2}{2r} \sqrt{\frac{10-4\sqrt{2}}{2}} = \frac{v^2}{2r} \sqrt{5-2\sqrt{2}} \\
&= \frac{v^2}{2r} \sqrt{2.172} \quad (\text{B})
\end{aligned}$$

8. For same angular speed ω

$$\omega = \frac{2\pi n}{t_1}$$

n : fractional revolution number of revolution

For 'A' $n = 1/4$

$$\omega = \frac{2\pi(1/4)}{t_1} \quad \omega = \left(\frac{v}{r}\right)$$

$$\frac{v}{r} = \frac{\pi}{2t_1} \quad \dots\dots (i)$$

$$\text{For B car } t_1 = \left(\frac{\pi r}{2v}\right)$$

$$\omega = \frac{2\pi r}{t_2} = \left(\frac{\pi}{4t_2}\right) = \left(\frac{v}{2r}\right)$$

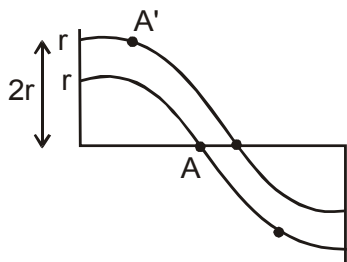
$$\frac{v}{r} = \left(\frac{\pi}{2t_2}\right)$$

So time interval for equal angular velocity

$$\Rightarrow \Delta t = \left(\frac{\pi r}{2v}\right) = t_1 = t_2 .$$



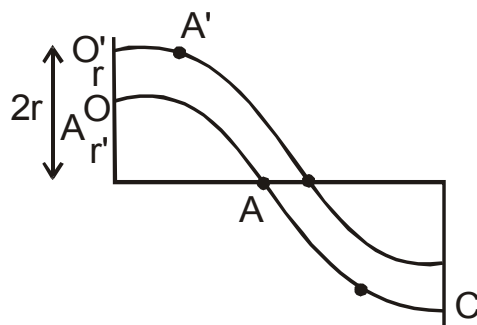
9. (A) Both have same linear speed at mean
 When A at A' than B at A'
 When A at B than B at B' When A at C then B at C'



(B) Friction for always towards centre to provide

sufficient centripetal force $\frac{mv^2}{r}$

(C) From above diagram



$$\text{For car A} \rightarrow O' \text{ to A } fs = \frac{mv^2}{r}$$

$$\text{For car B} \rightarrow O' \text{ to B' } fs' = \frac{mv^2}{2r}$$

$$\text{For A} \rightarrow A \text{ to C} = \frac{mv^2}{2r} = fs$$

$$\text{For B} \rightarrow B' \text{ to C'} = \frac{mv^2}{r} = fs'$$